$$R_g^2 = \frac{\int \rho(s)\, s^2\, dV}{\int \rho(s)\, dV}$$

Rg of Cube

$\rho(s) = \text{const.}$ so cancels

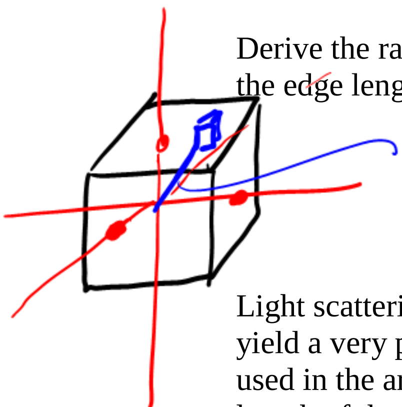
It is possible to synthesize nanocubes:

$s^2 = x^2 + y^2 + z^2$

http://www.sciencemag.org/cgi/content/full/298/5601/2176

$x \equiv y \equiv z$

Derive the radius of gyration of a uniform solid cube, expressing your result in terms of the edge length, L, of the cube.



s = distance from origin to volume element $dx\,dy\,dz$ at x, y, z.

$$\text{So } R_g^2 = \frac{1}{L^3} \int\!\!\int\!\!\int_{-L/2}^{+L/2} dx\,dy\,dz\,(x^2 + y^2 + z^2)$$

$$= \frac{L^2}{4} \text{ or } R_g = L/2$$

Light scattering (maybe better yet, small angle X-ray scattering) measurements would yield a very precise radius of gyration, without the sampling limitations of microscopy as used in the article above. Suppose such measurements give Rg = 70 nm, what is the edge length of the cube?

$$R_g = L/2 = 70 \text{ nm}$$

$$L = 140 \text{ nm}$$

What is the molecular weight of these cubes, assuming they are made of silver atoms?

$$\rho = 10.5 \text{ g/mL}$$

$$M = V \rho N_A = (140 \times 10^{-7} \text{ cm})^3 (10.5 \text{ g/cm}^3) \times 6.02 \times 10^{23}$$

$$\cong (1.4 \times 10^{-5})(10)(6 \times 10^{23})$$

$$\cong 2.7 \times 10 \times 6 \times 10^{8}$$

$$= 170 \times 10^{8} = \boxed{1.7 \times 10^{10}}$$