Intrinsic Viscosity Molecular Weight Average

The intrinsic viscosity average molecular weight is given by:

$$M_v = \left(\frac{\sum n_i M_i^{1+a}}{\sum n_i M_i} \right)^{1/a}$$

a) Write this expression in terms of weight fraction, w_i.

For a 2 component system: $M_V = \left(\frac{n_1 M_1 M_1^a}{n_1 M_1 + n_2 M_2} + \frac{n_2 M_2 M_2^a}{n_1 M_1 + n_2 M_2} \right)^{1/a}$ ($\frac{n_1 M_1}{n_1 M_1 + n_2 M_2} = w_i$) $\xrightarrow{\text{regeneralize}} M_v = \left(\sum w_i M_i^a \right)^{1/a}$

b) Starting from the definition of intrinsic viscosity in terms of the limiting value of the specific viscosity divided by concentration at low concentrations, derive the expression.

$[\eta] = \lim_{c \to 0} \frac{\eta - \eta_0}{c\eta_0} = K M_v^a$ (eqn. A) — two equations already discussed

Assume linear addition $\rightarrow \eta - \eta_0 = c_1 \eta_0 K_1 M_1^a + c_2 \eta_0 K_2 M_2^a + \ldots$

make into a summation notation: $\eta - \eta_0 = \eta_0 K \sum c_i M_i^a$ $\xrightarrow{\text{make into original form}}$ $\lim_{c \to 0} \frac{\eta - \eta_0}{c\eta_0} = \frac{K \sum c_i M_i^a}{c} \Big|_{c=0}$ ($c\eta_0$: total concentration)

Discussed previously in class $\left\{ c_i = \frac{g_i}{V} = \frac{n_i M_i}{V} \right.$. Now plug into for c_i and $c = \frac{\sum n_i M_i}{V}$

$$\lim_{c \to 0} \frac{\eta - \eta_0}{c\eta_0} = [\eta] = \frac{K \sum \frac{n_i M_i}{V} M_i^a}{\frac{\sum n_i M_i}{V}} = \frac{K \sum n_i M_i^{1+a}}{\sum n_i M_i}$$ (eqn B)

Compare eqn. A and B $\rightarrow$ $M_V = \left(\frac{\sum n_i M_i^{1+a}}{\sum n_i M_i} \right)^{1/a}$