Use Calculus to explore $(1-p)^{-1}$

It was shown that $\sigma_n = (1 - p)^{-1}$. Usually, we think about large values of p. This problem concerns small values of p and is just an exercise to remind you how we derive some limiting forms of important equations.

Use calculus skills to prove that:

$$\frac{1}{1-p} \cong 1 + p$$

for small p. Note: writing out a sum or saying magic words like "geometric series" does not constitute a *calculus-based proof.*

Let $y = \frac{1}{1-p}$ —identify easy point about which expansion makes sense→ $\frac{1}{1-p}\Big|_{p=0}$ + Taylor type terms

O.K. $\frac{1}{1-p} = \frac{1}{1-p}\Big|_{p=0} + \left(\frac{\partial}{\partial p}\frac{1}{1-p}\right)_{p=0} p + \left(\frac{\partial^2}{\partial p^2}\frac{1}{1-p}\right)\frac{p^2}{2!} + \cdots$

$= 1 + \frac{1}{(1-p)^2}\Big|_{p=0} p + \cdots$ OK enough

$\frac{1}{1-p} \cong 1 + p$