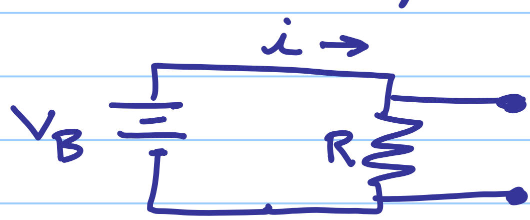
Basic electronics

Note Title 2/12/2008

Resistors: Carbon or metal film.

Consider a simple battery-resistor circuit



$$V_{meas} = V_B = iR$$

$$\text{units of } R = \frac{\text{volts}}{\text{amp}} = \frac{\text{joule}/\text{coul}}{\text{coul}/s} = \frac{\text{joule}\cdot s}{\text{coul}^2} = \frac{\frac{kg\cdot m^2}{s^2}\cdot s}{\text{coul}^2}$$

$$= kg\, m^2\, s^{-1}\, C^{-2}$$

$$= \text{"ohms"}$$

Replace battery with AC current source

$i = i_0 \cos\omega t$

$$V_{meas} = i_0 R \cos\omega t$$

note: $\omega = 2\pi f$

Voltage is in phase with current.

Capacitors — let's make some
1. Leyden jar
2. foil - tape - foil

Real capacitors can be "electrolytic", "oil", "polypropylene", "ceramic"

Basic equation: $C = \frac{Q}{V}$ = how much charge can it store for a given voltage.

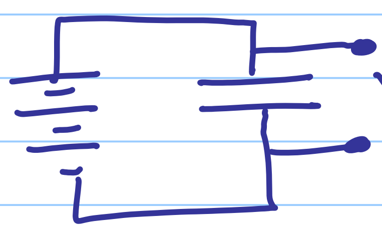
Symbol ⊣⊢

Unit = coul²/joule = FARAD

Again, consider a battery circuit

$i = 0$

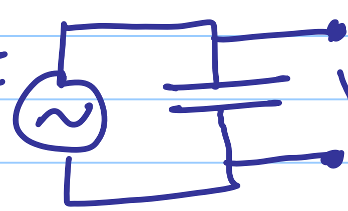
V_B



$V_{meas} = V_B$

Again, replace battery with AC current source:

$i = i_0 \sin \omega t$



V_{meas}

Current?
Well, electrons flow to the cap, then away from it.
No net flow, but oscillations

However, we expect $i = 0$ at $\omega = 0$:
capacitor charges to the point where it just cancels the source.

What is V_{meas}?

$$V_{meas} = \frac{Q}{C} = \frac{1}{C}\int i \cdot dt$$

$$= \frac{1}{C}\int i_0 \sin \omega t$$

$$= \frac{i_0}{\omega C} \cos \omega t$$

$$= \frac{i_0}{\omega C} \sin(\omega t - 90°)$$

Capacitive Reactance: like resistance

$$V = IR$$

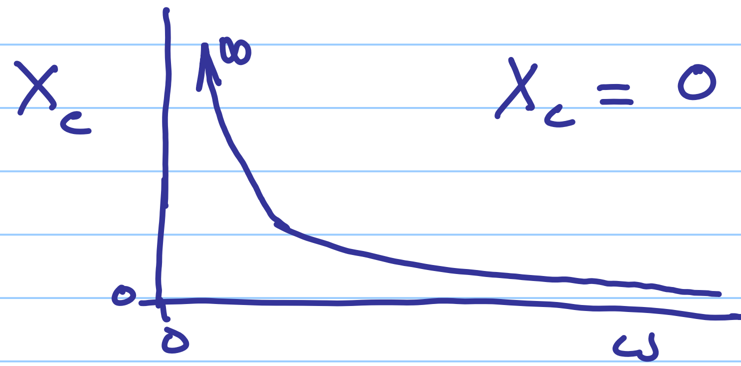
$$\Rightarrow V = I X_c$$

$$X_c = \frac{V}{I} = \frac{\frac{i_0}{\omega C} \sin(\omega t - 90°)}{i_0 \sin(\omega t)}$$

The concept of reactance averages over cycles. If we neglect the phase and we see:

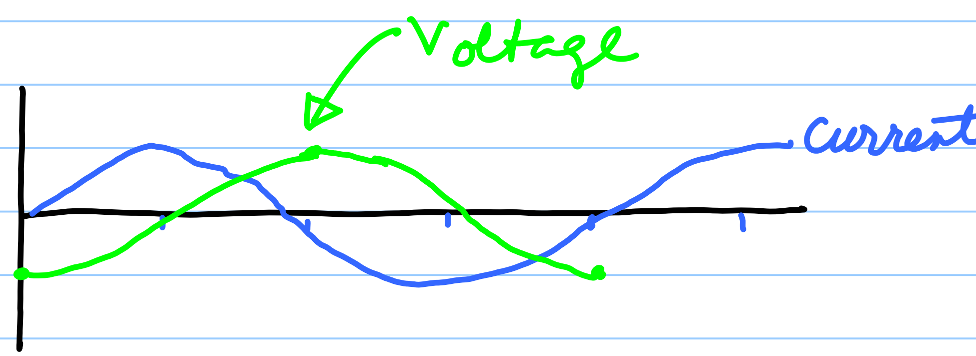
$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$X_c = \infty \text{ at } \omega = 0$$



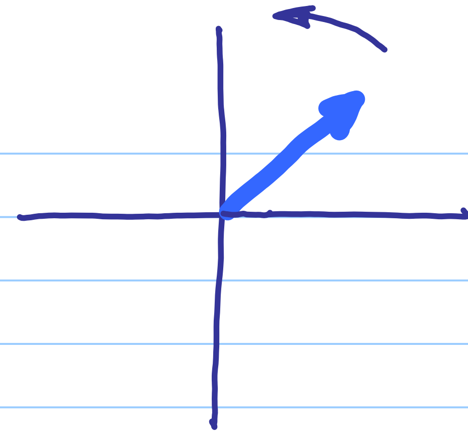
$$X_c = 0 \text{ at } \omega = \infty$$

In detail, we do need to know about the phase difference.



Current is required to build the voltage, so we say voltage lags current.

Phasor diagrams

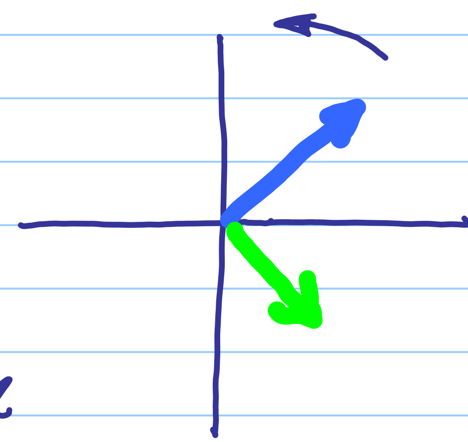


let the vertical component of the green line represent current.

As the arrow rotates (counterclockwise, from $\theta = 0$ to 90 to 180 etc) the vertical component traces out a sine wave. Now, lets add a green arrow to represent the voltage across the capacitor. What should its orientation be?

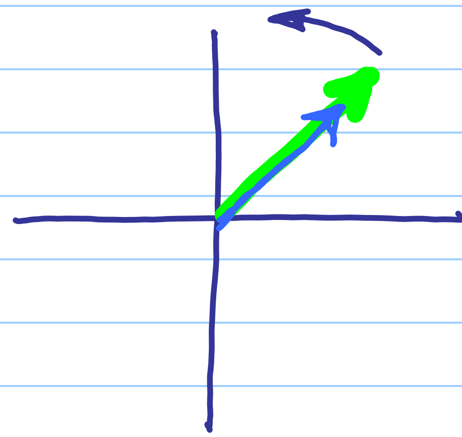
It should lag by 90°.

Phasor diagram for capacitor: if i is maximum $t = 0$, $V_{capacitor}$ will hit a maximum one-fourth of a cycle later. V lags i.



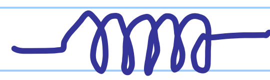
It is easy to see that the phase diagram for a resistor should have the green and blue (voltage & current) arrows held parallel.

Phasor diagram for resistor: voltage & current are in phase.



Inductors (Coils, "Chokes")

Inductors generate a voltage opposite the imposed according to acceleration of current.

Symbol is  because the induced EMF (electromotive force) arises from current in the coil. (See law of Biot-Savart in your Physics book).

Most inductors are "air core" or "iron core."

$$V_{inductor} = -L \frac{di}{dt}$$

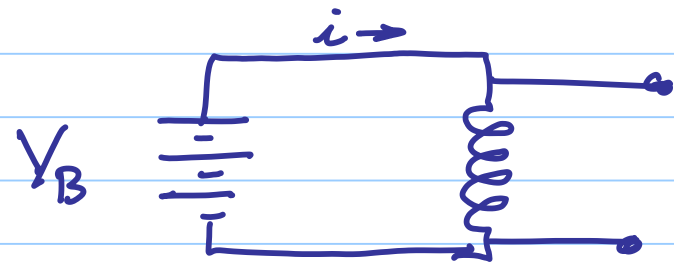
L — inductance

Units are

$$\frac{\frac{joule}{coul}}{\frac{coul}{s^2}} = \frac{joule}{coul^2 \cdot s^2}$$

= "Henry"

Consider the battery circuit

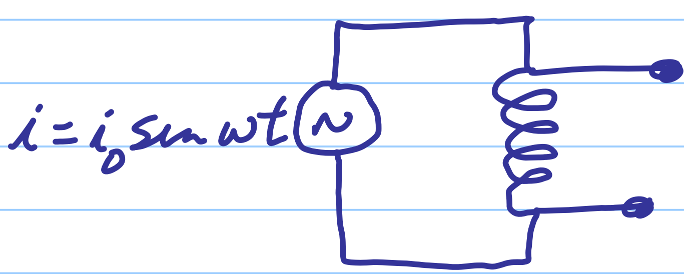


$V_{meas} \sim 0$!

Why is $V_{meas} = 0$? Because the resistance offered to a DC source by a piece of wound-up wire is almost zero. Thus, the battery will short out. If it is a powerful battery, so much current might flow that the wires and/or inductor could melt!

So, the inductor is opposite the capacitor: low resistance to DC.

Consider the circuit with an AC current source



$V_{meas} = -L \dfrac{di}{dt}$

The voltage is out of phase with the current:

$$V = -L \frac{d}{dt}(i_o \sin \omega t)$$

$$= -L\omega i_o \cos \omega t$$

$$= -L\omega i_o \sin(\omega t + 90^\circ)$$

Inductive Reactance: ignoring the phase shift, what is the analog to resistance?

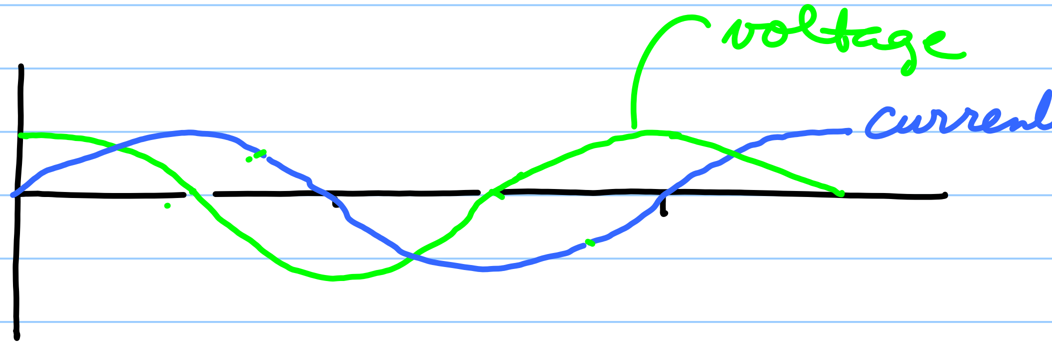
$$V = i X_L$$

$$\Rightarrow \quad X_L = \frac{V}{i} = \frac{-L\omega i_o \sin(\omega t + 90^\circ)}{i_o \sin \omega t}$$

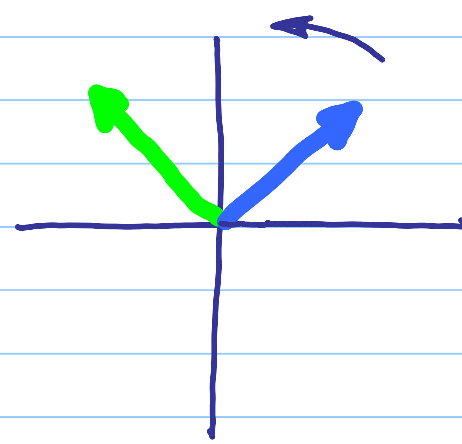
$\Downarrow$ neglect phase & sign

$$X_L = \omega L$$

The phase shift is opposite in sense to that arising from a capacitor:



It's a positive cosine wave, not a negative one: The voltage leads the current by 90°.



Phasor diagram for an inductor: When the rate of change of current flow is maximum (e.g. at $t=0$) so is the voltage across the inductor. So... maximum voltage occurs at minimum current. V leads i.

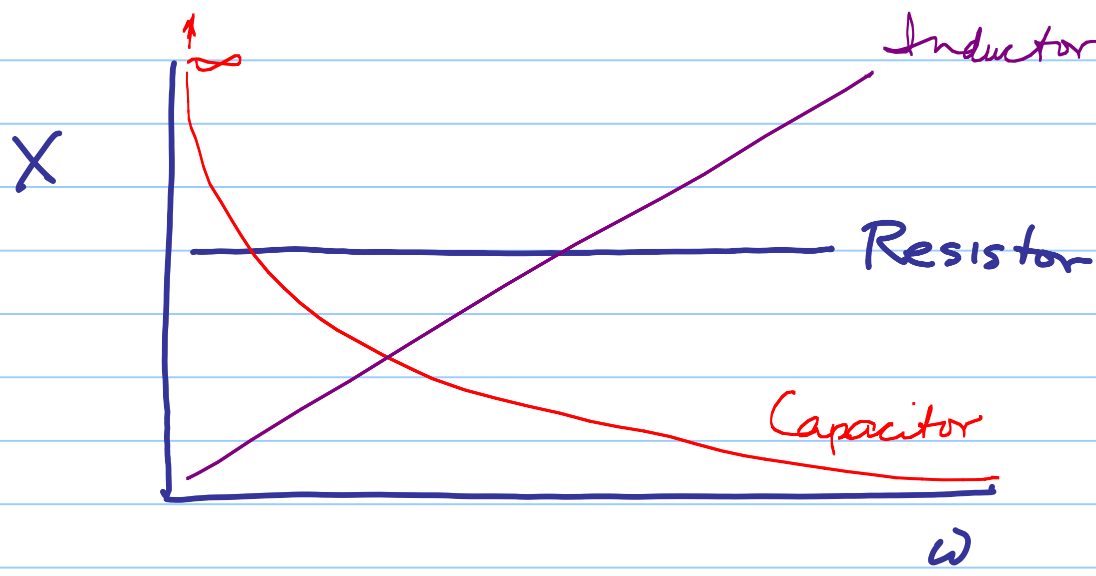
Mechanical Analogs of electrical elements

Resistor is like a friction element : $F = f v$

Capacitor is like a spring : $F = k(\Delta x)$

Inductor reminds us of mass : $F = m a$

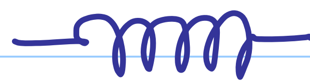
Summary of Reactance



Series Combinations

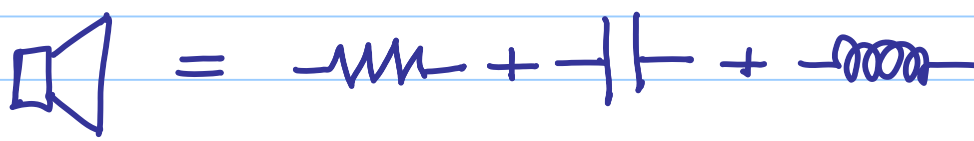
Series combinations of elements are important in several applications

Also, most real-world elements are not pure: an inductor always presents a little DC resistance.

e.g. 

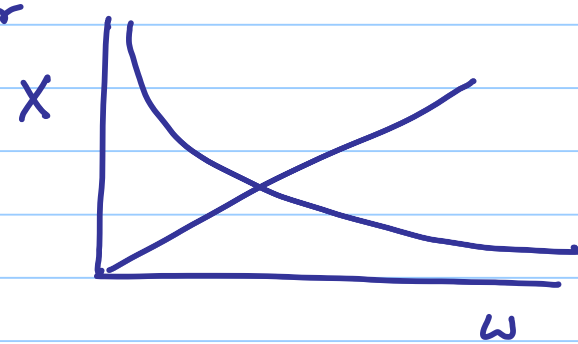
really equals 

The same is true of transducers, such as loudspeakers:



Where the capacitive term in this case comes from the pliant speaker surrounds, the resistive and inductive terms come from thin wire wrapped up to make a voice coil. There is also a mechanical mass that ought to behave like an inductance.

Look again at the plot of X vs. ω for capacitor & inductor



It isn't hard to imagine that a series combination of capacitor & inductor would have minimum total reactance at some medium frequency.

The series L-C circuit in fact resonates.

Energy is stored in the capacitor and inductor alternately. The overall load looks almost resistive as a result of +90° and −90° phase terms cancelling.

The overall resistance posed is just due to the length of wire used to make the inductor.

So.... if one looks at the voltage across either element (L or C) separately, there is a big AC voltage as the phasor rotates about. If one looks at the voltage of the series circuit, though, it is nearly zero.

Here is the phasor diagram. The yellow arrow represents the small DC resistance of the inductor. Mostly, the inductor & capacitor cancel each other out.