Fractals

Tuesday, July 08, 2008
1:16 AM

Why study fractals

Fractals Generator

rm2146_MandelbrotCoast
Mandelbrot's original *Science* article (1967)

Mandelbrot on how the paper came to be, its importance, etc.

Reference: Rubinstein & Colby
Polymer Physics Ch. 1

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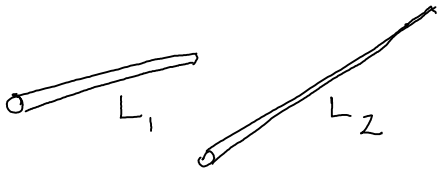
Review 4010 material

Mass ~ size ^{d_f}

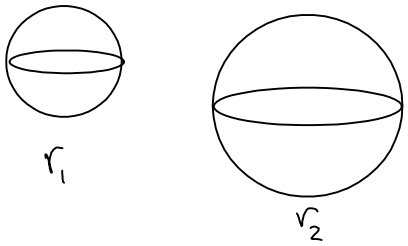
DNA
PHIC
PBZT

$d_f = 1$



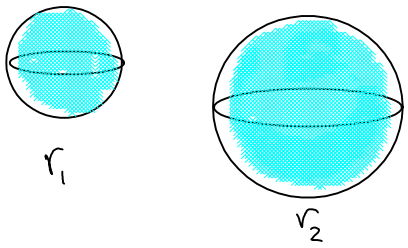
liposomes

$d_f = 2$

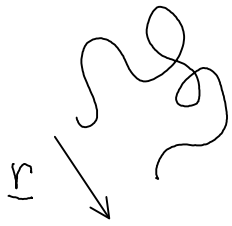


latex spheres
silica spheres

$d_f = 3$



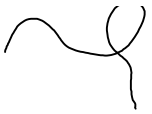
$\langle r^2 \rangle =$ _____



bad solvent — attractive

ideal solvent — neutral

good solvent — repulsive

good solvent  repulsive

? How can we arrange neutral interactions?

- in solutions
- in melts
- in gels

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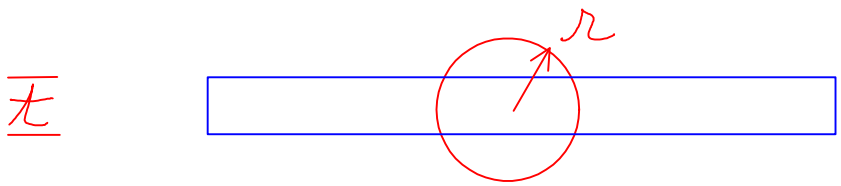
How far can we push this idea of dimensions?

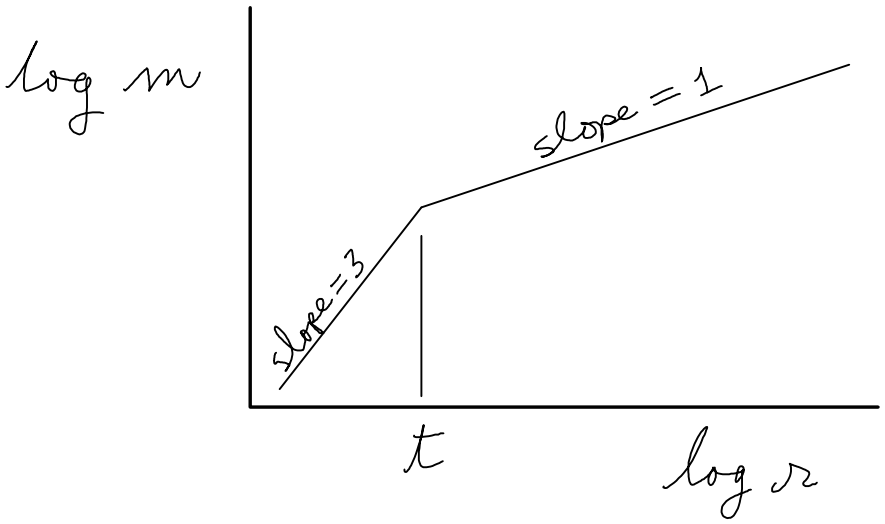
Consider a rod circumscribed by a sphere of radius r



Define: m = mass of rod encased by the sphere of radius r

When $r < t$, $m \sim r^3$

When $r > t$, $m \sim r^1$



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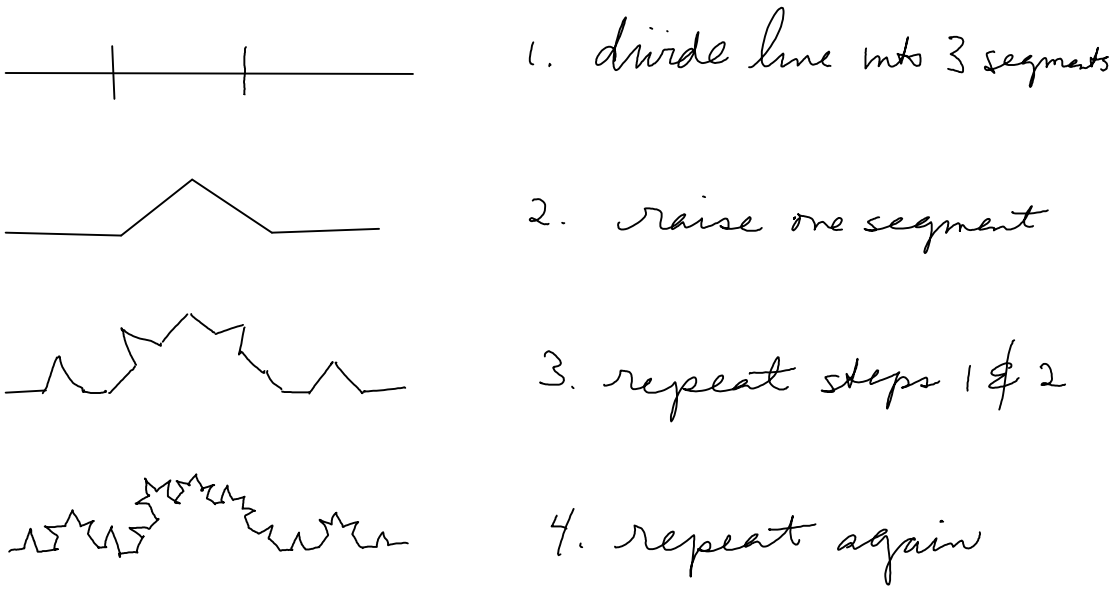
A more sophisticated look----the history & beauty of regular fractals

Koch curves



Notice how the middle bump stays…basic shape stays the same. Looks the same at all scales! But much more "regular" and beautiful than wiggly polymers.

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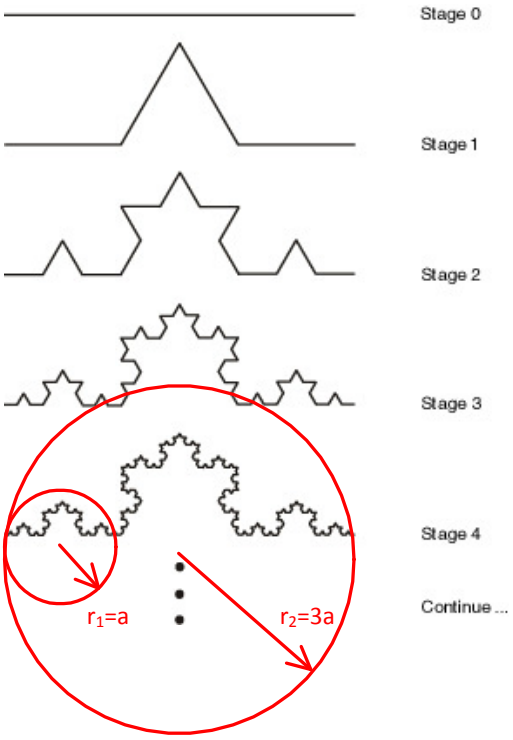
[Koch curves and snowflakes Wiki page]

Here is a better drawing of the Koch curves. I have added circles at radius r_1=a and r_2=3a



Pasted from <http://www.bing.com/images/search?q=koch+curves&view=detail&id=71FCC2FF82F4AB5F57D6BDA56F6233AB103A0EB3&first=0&qpvt=koch+curves&FORM=IDFRIR>

It works out that the mass in the bigger circle grows faster than the circle radius itself.

Can you see the connection to "How long is the coast of Britain?" (Answer: Depends on the size of the ruler. As we add more zigzags, the ruler here is growing smaller.)

$$m \sim r^{d_f}$$

$$m_1 \sim r_1^{d_f} = a^{d_f}$$

$$m_2 \sim r_2^{d_f} = (3a)^{d_f}$$

It turns out $m_2 = 4 m_1$

$$m_2 = 4 m_1 = 4 a^{d_f} = (3a)^{d_f}$$

take logs ⇒ rightmost equation

$$\log 4 + d_f \log a = d_f \log 3 + d_f \log a$$

$$d_f = \frac{\log 4}{\log 3} = 1.262$$

So…. Mass ~ size$^{1.262}$

Although the snowflake is drawn with just lines, its mass grows faster than a line.

Similarly, the length of the coast of England grows faster than the size of the object used to measure it decreases.

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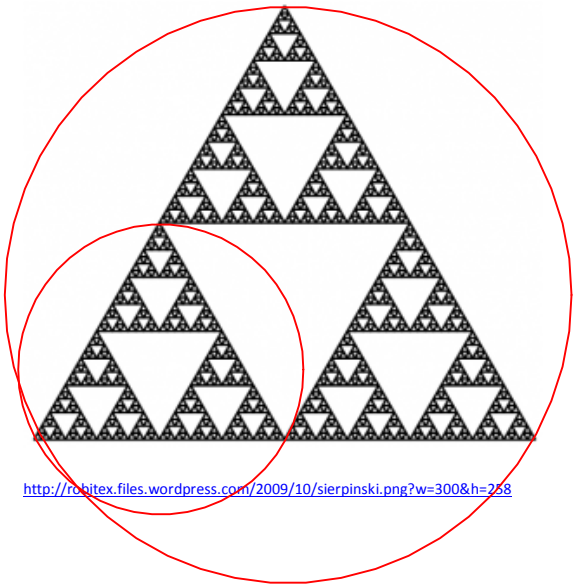
Another classic regular fractal is the Sierpinski gasket.



http://robitex.files.wordpress.com/2009/10/sierpinski.png?w=300&h=258

So, in this case there is three times as much mass in the circle that has twice the radius.

$$d_f = \frac{\log 3}{\log 2} = 1.585$$

This object is closer to a 2D thing than the Koch snowflake

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Freehand discussion of DLA, RLA, DLCC etc.

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SAXS and USAXS on aerogels

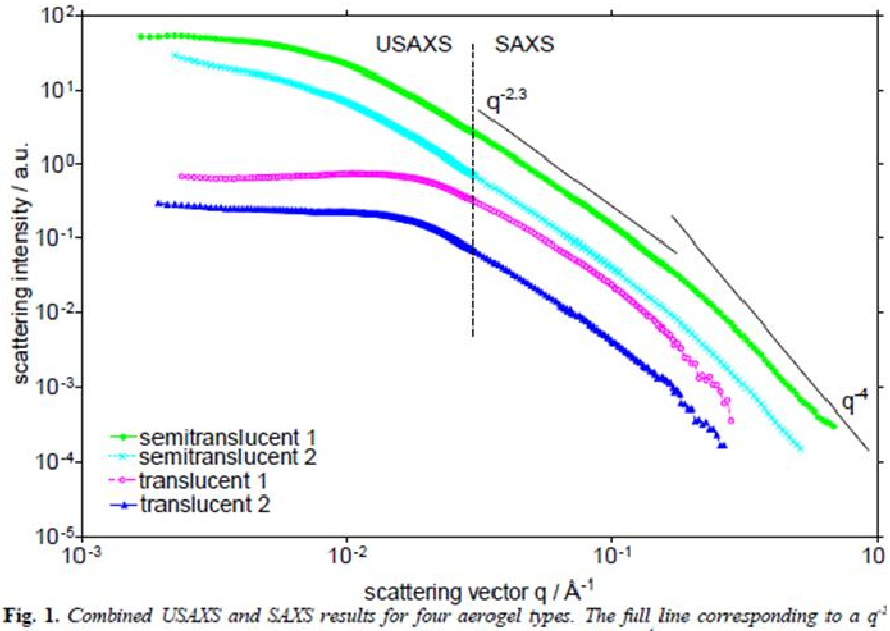
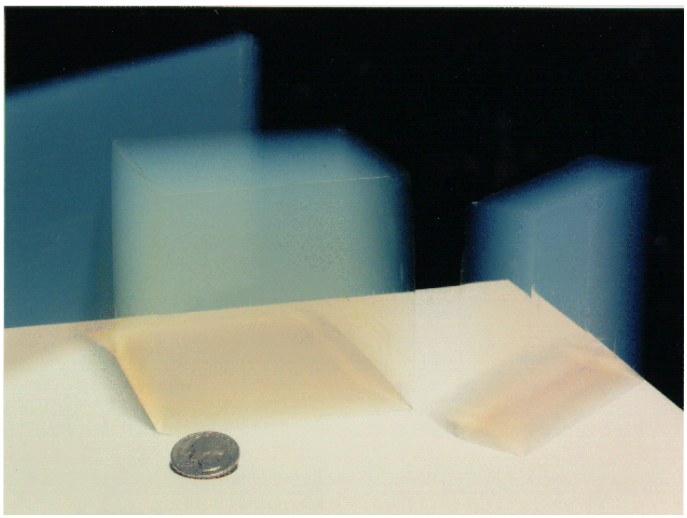
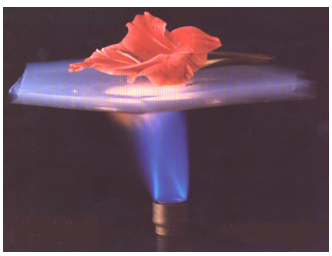


Fig. 1. *Combined USAXS and SAXS results for four aerogel types. The full line corresponding to a q^{-1} decay of the scattering intensity indicates the mass fractal dimension, while q^{-4} represents the Porod slope.*



Pasted from <http://cgi.fiu.edu/expnuc/aerogel/aerogel_1.gif>



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